# Acceleration of Iterative Solver for Electromagnetic Analysis using GPU/MIC

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**The mixed precision variable preconditioning (VP) Krylov subspace method is implemented on Graphics Processing Unit (GPU) and Many Integrated Core (MIC) architecture, and the linear system obtained from an electromagnetic analysis is solved by the method. In recent year, high-performance multi/many core computer architecture can be cheap and easily available, and the simulation code must be parallelized by using parallelization API. Although the ordinal program code that developed on CPU can be implemented on MIC without transcribing, GPU programming cost using CUDA becomes very high. In the present study, the performances of the mixed precision VP Krylov subspace method on GPU and MIC are compared by solving the linear system obtained from electromagnetic analysis discretized by edge element. The results of computation show that the communication cost of MIC is much higher than that of GPU.**

*Index Terms*—**Electromagnetic analysis, Linear system, GPGPU, Many Integrated Core Architecture, High performance computing.**

## I. Introduction

RECENTLY, a clock frequency of CPU has gone as far as it can go, and a multi-core processor and an accelerator such as Graphics Processing Unit (GPU) are adopted for high performance computing calculations [1], [2]. Therefore, the simulation code should be parallelized by Compute Unified Device Architecture (CUDA) or parallelized API such as Message Passing Interface (MPI). Generally, GPU programming cost using CUDA becomes very high.

Many Integrated Core (MIC) architecture appears on the scene of high performance computing, and about 60 cores are implemented on unit device. Since these cores are x86 architecture, the ordinal program code that developed on CPU can be implemented on MIC without transcribing, and very easy to parallelized by using OpenMP [3].

As is well known that a singular coefficient matrix is obtained from discretizing electromagnetic phenomena using FEM with edge elements. And a singular solution should be calculated. Frequently, the preconditioning Krylov subspace methods are adopted for the problem. In the previous study [3], [4], we have implemented the Variable Precondition (VP) Krylov subspace method with mixed precision on GPU and MIC, and investigated the performance of the method. However, the performances of both devices were not compared.

The purpose of the present study is to implement the VP conjugate gradient method with Jacobi Over Relaxation (JOR) method on GPU and MIC, and the performances of both devices are investigated.

## II. GPU and MIC

In the present study, two types of supercomputer at Center for Computational Science, University of Tsukuba are used for evaluations. HA-PACS is constructed by 332 nodes, and the peak performance of the system is 1.166 PFLOPS. Two multi-core CPUs and four GPUs are implemented on unit node. COMA is constructed by 393 nodes, and two multi-core CPUs and two MICs are implemented on unit node.

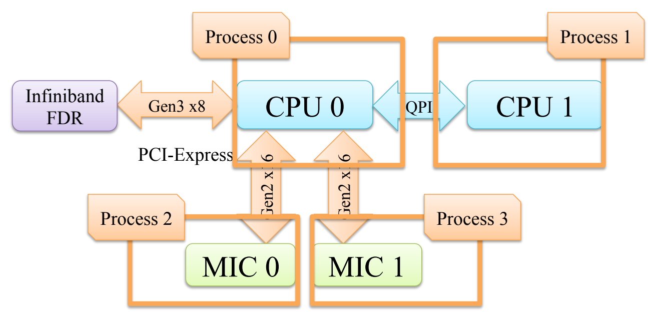


Fig. 1. The schematic view of the parallelization policy on multi-MIC using the symmetric mode.

Multi-GPU and multi-MIC are used for evaluations. In the GPU evaluation, two GPUs are employed in unit node, and each node is connected by Infiniband Network. In addition, each process is assigned to unit GPU. On the other hand, the symmetric mode is employed for multi-MIC calculation, and the concerted calculation between CPU and MIC is implemented. Processes are assigned to two CPUs and two MICs in the unit node, and 16 nodes are used for evaluation (see Fig. 1).

## III. Problem and VP Krylov Subspace Method

In this study, the Problem 20 in testing electromagnetic analysis methods (T.E.A.M.) Workshop is employed for the benchmark. Though the original problem is a nonlinear problem, the value of relative magnetic permeability is fixed as 200 so that the problem becomes a linear problem. The number of edge element of the problem is 27,549,822 and the dimension size of the coefficient matrix is 1,709,028. Note that the coefficient matrix becomes very sparse matrix, and only 42 nonzero elements include in unit column. From this reason, Compressed Row Storage (CRS) and Jugged Diagonal Storage (JDS) are employed for MIC calculation and GPU calculation, respectively.

The original VP Krylov subspace method has two nested iterations for GCR and variable preconditioning for GCR are called as outer-loop and inner-loop, and the method has the sufficient convergence condition. The residual of the problem converges if the relative residual norm of inner-loop satisfies less than one in each steps. Besides, this sufficient condition is derived from the monotonicity of GCR method [5]. By taking into account of above character, we extend the method with mixed precision that uses a single precision operation for inner-loop and a double precision operation for outer-loop, and the method is called as Mixed Precision VP Krylov Subspace method [4]. Additionally, the conjugate gradient (CG) method is adopted for outer-loop and Jacobi over relaxation method for inner-loop because of the parallelization performance.

## IV. Results and Discussions

Let us investigate the total execution time of VPCG with JOR on multi-GPU and multi-MIC. As we mentioned above, a single precision operation and a double precision operation are used for inner and outer-loop, respectively. The maximum iteration number for inner-loop is fixed as two, and termination conditions for inner and outer-loop are fixed as $10^{-1}$ and $10^{-8}$.

The total execution times that include computation time, inner-loop communication time and out-loop communication time are shown in Fig. 2 and Fig. 3. We can see from these figure that the total execution time of multi-GPU is much better than that of multi-MIC. However, the calculation times of the multi-MIC are much better than that of multi-GPU. That is to say the communication times between the devices are dominated in case of multi-MIC. This result is caused by the architectural issue of MIC. In the GPU cluster, data communicate by mainly CPU, whereas the MIC is contracted for the most of all the communication in the MIC cluster. From this reason, the communication performance is degraded. Thus, the communication hides technique is necessary for multi-MIC calculation.

These results also indicate that there exists the optimal number of processes. The execution time decreases as the number of process increases at the first time. However, the execution time change to the increasing cases. This result caused by shortage of the dimension size of the matrix.

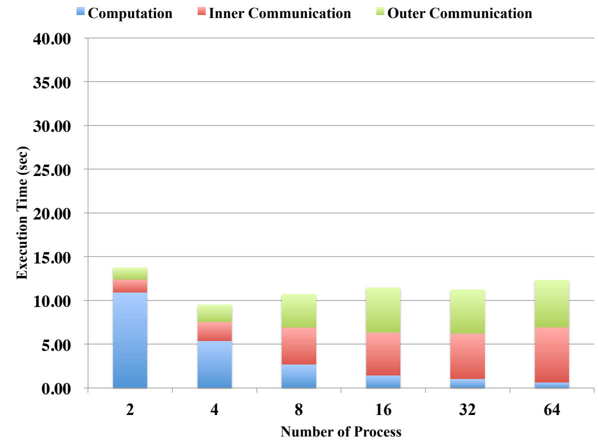


Fig. 2. The total execution time of VPCG with JOR on multi-GPU.

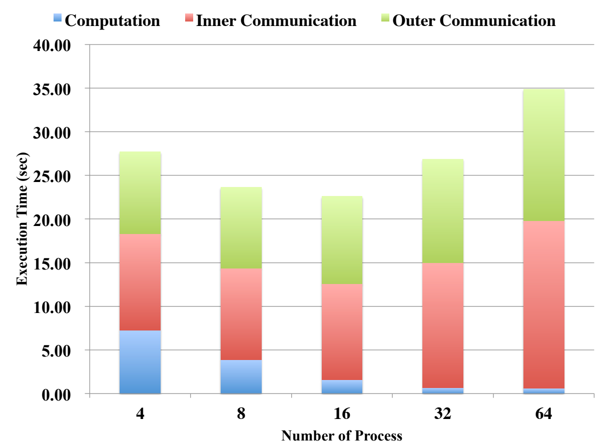


Fig. 3. The total execution time of VPCG with JOR on multi-MIC using the symmetric mode.

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